mapping of the domain mentioned into the domain of Fig. 1b is not accomplished by the simple function (1) in this case but by using Jacobi and Weierstrass elliptic functions [6], or by using trigonometric functions in the particular case  $H \rightarrow \infty$ . The Keldysh-Sedov formula again yields the solution of the problem in principle, but calculations and numerical computations may turn out to be quite awkward.

#### NOTATION

C, C', arbitrary constants; F, complex velocity in the  $\zeta$  plane; G, a quantity introduced in Eq. (20); H, dimensionless height of the bed; h, height of the jet; I, integral in Eq. (19); L, half the dimensionless spacing between adjacent jets; p, pressure; Q, gas discharge into the jet; r, r<sub>1</sub>, r<sub>2</sub>, functions defined in Eq. (14); U, complex velocity; u, gas filtration velocity; u<sub>x</sub>, minimal fluidization rate; v, excess velocity due to jet gas injection; x, y, dimensionless coordinates;  $z = x + iy; \alpha$ , hydraulic drag coefficient;  $\beta$ , a parameter introduced in Eq. (20);  $\Gamma$ , a parameter introduced in Eq. (16);  $\gamma$ , dimensionless height of the jet base above the gas distributing grid;  $\xi = \xi + in; \xi, \eta$ , coordinates in the  $\zeta$  plane;  $\lambda$ ,  $\lambda_1$ ,  $\lambda_2$ , angle functions introduced in Eq. (14);  $\Phi$ , complex potential;  $\varphi$ , potential;  $\psi$ , harmonic conjugate function to  $\phi$ ; the degree superscript refers to quantities characterizing the unperturbed state of the bed.

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### SECTIONALIZED FLUIDIZED-BED EQUIPMENT FOR

### SOLUTION GRANULATION

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A mathematical model is presented for solution granulation; the model has been tested by experiment. The model indicates that sectionalized equipment is suitable.

We have previously [1] examined the advantages of two-stage (countercurrent and crossflow) sectioning in a fluidized-bed system in which heat of reaction is released and this heat is removed by evaporation of input water. The sectionalization in that case gives a considerable improvement in the specific throughput. This system has now been introduced in a commercial equipment designed by the Urals Chemical-Research Institute and intended for treating sodium sulfate.

However, the heat- and mass-transfer processes occurring in solution granulation are very different from those occurring in many fluidized beds. Here we employ mathematical simulation to examine the effects of sectioning on the performance parameters of a fluidized bed intended to handle solutions. The results show that sectioning improves the throughput without increasing the energy consumption.

The model presupposes ideal mixing in the fluidized bed, so the temperature and effective water content of the granules are taken as identical throughout the height of the bed.

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Fig. 1. Comparison of the calculated temperature A (°C) and the observed temperature B at the outlet on fluidized-bed granulation: 1) gas temperature at outlet; 2) material temperature.

As all the granules are exposed to the solution simultaneously, the bed at any instant contains particles coated with solution (wet particles). These wet surfaces evaporate water. On the other hand, many of the particles have dry surfaces and do not evaporate water.

The gas in the system moves in absolute-displacement mode.

The model is described by the following first-order nonlinear ordinary differential equations (1)-(5):

$$\frac{dz}{dy} = \frac{\beta U}{g_c} \left( P\left(T\right) - P\left(y\right) \right). \tag{1}$$

Equation (1) represents the variation in gas water content over the height,

$$\frac{d\theta}{dy} \left( c'_c + z \right) + \left( \frac{\alpha F}{g_c C_n} + \frac{dz}{dy} \right) \left( \theta - T \right) = 0.$$
(2)

Equation (2) describes the variation in gas temperature, which is due in part to heat lost from the dry particles and heat taken up with the vapor from the coated particles

$$\frac{a}{dt} [G(W)Tc_1(W)] + g_c \{c'(z_0)\theta_0 - c'(z_H)\theta_H - r_1(z_H - z_0) + c_5g_5\tilde{T} + \mu [g_1(W) - g_1(W_0)] - \tilde{g}_1(W)c_1(W)T + g_1(W_0)c_1(W_0)T_0\} = 0.$$
(3)

Equation (3) is the heat-balance equation, and it incorporates the input heat and heat loss in all forms

$$\frac{d}{dt} [G(W)] + g_c \{ \tilde{g}_1(W) - \tilde{g}_1(W_0) + g_1(W) - g_1(W_0) \} = 0.$$
(4)

Equation (4) is the water-balance equation for the particles

$$-\frac{dG}{dt} + (g_1 - \tilde{g}_1)g_c = 0.$$
 (5)

Equation (5) describes the time dependence of the amount of material in the fluidized bed.

We can solve (1)-(5) by specifying the temperature and water content of the gas at the inlet to the fluidized bed together with the temperature and concentration of the incoming solution, which gives the temperature and water content of the material and of the gas at the outlet for a given instant, which thus serves to define the proportion of wet particles. It is possible to determine the outlet parameters in the steady state and in the transient state, as well as the duration of the transient state, which is important in the design of automatic controls.

System (1)-(5) is solved as follows. Equations (1) and (2) are integrated by the Runge-Kutta method, while ordinary inexplicit difference schemes are used for (3) and (4), and an explicit difference scheme for (5).



Fig. 2. Effects of fraction of gas  $n_2$  supplied to the second section on: a) temperature; b) water content of material at the output from: I) first section; II) second section; the numbers on the curves are the proportions (%) of the solution supplied to the first section.

The job was handled with a BÉSM-6 computer; the suitability of the model was checked against data from the Urals Chemical-Research Institute on the granulation of concentrated phosphate fertilizers. In the experiments, the temperature of the material in the fluidized bed was in the range 60-95°C, i.e., below the boiling point of the solution. Such a process may be called a low-temperature one. In that case, the film of solution on the granules evaporates mainly by heat supply from the gas.

We also examined the high-temperature granulation mode, in which case the particles have temperatures above the boiling point of the input solution. Much of the heat involved in evaporating the solution then comes from within the granules. An example of this is an industrial system for granulating sodium dichromate, for which data were available [2].

Figure 1 shows that the theoretical and experimental gas-temperature and composition results are in satisfactory agreement, so the model is a good fit to the actual process.

This model was used in examining two-stage granulation; the working parameters were taken as close to those for one of the modes of granulation used with concentrated phosphate fertilizer in a single-stage equipment, namely, specific fluidization gas flow rate 5634 kg/ $m^2/h$ , gas temperature at inlet 380°C, and water content 19.6 g/kg of dry gas. The input so-lution was at 40°C and contained 69% water. The solution was used at the rate of 0.1764 kg/kg of dry gas. Dry material of temperature 96°C then emerged from the one-stage system.

The working conditions for the two-stage system were taken as follows: The total amount of fluidization gas was divided into two flows, which were passed in parallel into the first and second sections (along the direction of solid motion). The bed in each section was of the same height, the height being that used for the one-section system.



Fig. 3. Additional amount of solution ( $\Delta m$ , %) that can be used in a two-section apparatus: 1) for the parameters given in the paper; 2) for  $T_{\rm H} = 152^{\circ}$ C.

We examined various distributions of the gas flows between the sections and various solution input flows. It was assumed that the solid product from the drying passed in sequence from the first section to the second. The results are shown in Fig. 2, for sections I and II. These results will now be discussed in sequence. Figure 2, part Ia shows the amount of gas supplied to the second section with percent of the total flow as abscissa, while the ordinate is the temperature of the material at the outlet from the first section. The curves relate to different amounts of solution input to the first section. The amounts of solution supplied to the second section are shown directly on the curve. If the gas flow to the second section is increased (and that to the first section is correspondingly reduced), the temperature at the outlet of the first section falls to the horizontal line corresponding to 61°C, which is the adiabatic saturation temperature for the fluidizing gas. At that temperature, the material may be moist on emerging from the first section. The percent of water content WH, in the material can be derived from Fig. 2, part Ib. This part of the figure shows that modes giving an advantage in throughput in the two-stage system allow the water content at the outlet from the first section to be as high as 10%, which eases the working conditions in the transfer system. If the gas input to the first section is very small, the temperature of the material at the outlet falls somewhat further and ultimately tends to the temperature of the input solution.

Figure 2, part IIa shows the temperature at the outlet from the second section for various gas and solution inputs; section IIb of Fig. 2 shows the water content of the material leaving the second section.

We now consider an example. Let the amounts of solution pass to the first and second sections be identical at 50% of the total input; the curve ADBC represents the variation in the temperature of the material at the outlet from the first section as the gas flow is varied; if the second section receives less than 58% of the gas (the first section receives over 42%), the outlet temperature rises above the adiabatic saturation temperature (branch ADB). Then Fig. 2, part IIb shows that the material emerging from the first section is dry. If the second section receives more than 58% of the gas, the material emerges from the first section at the adiabatic saturation temperature  $T_{\alpha}$  and is moist (branch BC). Part Ib of Fig. 2 shows that the water content of the material at the outlet from the first section increases rapidly as the gas flow to that section is reduced and at point C attains the water content of the input solution, i.e., 69%.

We now consider the operation of the second section for the same case. Part IIa of Fig. 2 shows that if the second section receives less than 45% of the gas (branch AD), moist material emerges from the second section (line AD in Fig. 2, part IIb).

Over section DB, the second part of the apparatus receives more than 45% of the gas, while the input from the first section is dry material of temperature somewhat above  $T_{\alpha}$ , and therefore the output from the second section is dry material whose temperature is above that of material at the outlet from the one-section system, the value at point B being 122°C.

If the second section receives over 58% of the gas, the temperature of the material emerging from the second section begins to fall somewhat (branch BC), because the second section begins to receive material of increasing water content from the first section. The material still emerges dry from the second section. It is of interest to examine extreme modes of operation such that all the gas is supplied either to the first section (in which case the temperature of the material at the outlet is represented by point E in part Ia of Fig. 2) or else all the gas goes to the second section (temperature represented by point C in part IIa of Fig. 2). The equipment in that case degenerates into a one-section system with an outlet temperature of 96°C for the material, which is naturally the same for points E and C.

These curves show that there is considerable overheating of the outgoing material when the second section receives less than 25% of the solution, as the temperature can attain 240 or even 295°C. If the material must emerge dry from the second section, but without excess heating, one can add a certain extra proportion of the solution in order to reduce the temperature, perhaps as far as the temperature of material emerging from a one-section system (in our case 96°C). The amount of additional solution passed to the second section improves the throughput of the two-section system without increasing the energy consumption.

Figure 3 shows the amount of additional solution supplied to the equipment in relation to the proportion of gas  $n_2$  supplied to the second section. The peak of 13% occurs when the second section receives 16% of the gas, so sectionalization can increase the throughput appreciably.

The product from the one-section system must be dry, so the temperature of the material is kept above the adiabatic saturation temperature; the temperature of the outgoing gas with the bed depth used is essentially equal to the particle temperature in the bed. The sectionalization allows one to distribute the flows of solution and gas between the sections in such a way as to reduce the temperature of the material in the first section, since drying is completed in the second section, and therefore the outgoing gas temperature can also be reduced. Therefore, further heat can be extracted from the gas to evaporate the solution, which implies improved performance from the two-section system.

In practice, sectionalization is fairly simple, since one merely inserts a transverse baffle in the body of the vessel and supplies the solution separately to the two parts.

Although our numerical calculations and graph relate only to one form of material, the trends apply for other products, particularly in high-temperature granulation.

# NOTATION

 $\theta$ , z, g<sub>c</sub>, c'(z), c'<sub>c</sub>, temperature (°C), moisture content (kg/kg) of fluidizing gas, mass flow rate (kg/m<sup>2</sup>•h) of dry air, specific heats (dimensionless) of fluidizing gas and of dry air referred to specific heat at height y;  $\theta_0$ , z<sub>0</sub> and  $\theta_H$ , z<sub>H</sub>, parameters at the inlet and outlet, respectively; T<sub>0</sub>, g<sub>1</sub>(W<sub>0</sub>), c<sub>1</sub>(W<sub>0</sub>);  $\tilde{T}_1$ , g<sub>5</sub>, c<sub>5</sub>, temperature (°C), mass flow rate (dimensionless) referred to g<sub>c</sub>, and specific heat (dimensionless) of input to bed referred to specific heat of vapor, temperature (°C), mass flow rate (dimensionless), and specific heat of water (dimensionless) referred to specific heat of vapor; T,  $\tilde{g}_1$ (W), temperature (°C), mass flow rate (dimensionless) referred to g<sub>c</sub> at the outlet; G(W), c<sub>1</sub>(W), amount of moist material per m<sup>2</sup> of grid (kg/m<sup>2</sup>) and specific heat (dimensionless) of moist material referred to specific heat of vapor; H, F, U, P(T), P(y), bed height (m), total surface area of particles per unit volume (m<sup>2</sup>/m<sup>3</sup>), surface area of moist particles per unit bed volume (m<sup>2</sup>/m<sup>3</sup>), equilibrium partial pressure of water vapor above the solution at T, °C (abs. atm), partial pressure of water vapor at height y (abs. atm);  $r_1$ ,  $\mu$ ,  $\alpha$ ,  $\beta$ , latent heat of evaporation referred to specific heat of vapor (deg), heat of hydration per kg of water referred to specific heat of vapor (deg), heat-transfer coefficient (kcal/m<sup>2</sup>•h•deg), and mass-transfer coefficient (kg/m<sup>2</sup>•h•atm).

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AXIAL DISPLACEMENT OF SOLID PHASE IN A CONTAINED FLUIDIZED BED

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Thermal labeling has been used to examine axial heat transport (solid-phase transport) in a fluidized bed with low-volume packing for columns of diameters 15, 30, and 70 cm.

Many low-volume retaining systems [8, 9, 11, 12] such as horizontal grids, perforated plates, gridded cylinders, and wire spirals can adversely affect the displacement of the solid material, because they break up the gas bubbles and thus reduce the rise speed considerably [2, 17].

If such devices are to be used to advantage, especially when rapid particle motion is to be combined with high heat-transfer performance, it is necessary to know the laws governing the motion of the solid phase and the heat, particularly as regards effects of system scale.

The vertical particle movement in a free bed is similar to that in a contained one, in that there is circulation due to particle transport in the surfaces of rising bubbles, together with exchange of particles between the surfaces of bubbles by diffusion. When these bubbles flow around fixed elements in the containment, there is additional transfer, particularly due to displacement of particles from the surface of the bubbles.

Therefore, the transport of the solid (heat) in a fluidized system can be considered as a combination of convection and diffusion [10]. The available immobile packing accentuates the diffusion and tends to suppress the circulation. Measurements show that the gas nonuniformities in such a system are much smaller in scale and more uniformly distributed in space [2, 17], while the particle motion is close to the diffusion type [1, 18]. It would seem [13] that the radial displacement in such a system is extremely close to diffusion type.

As all the particles remain within the system for times much exceeding the time corresponding to descent through the bed, the transport may be considered of diffusion type and

Powder material	d <sub>av</sub> , mm	u <sub>0</sub> , cm/sec	p <sub>4</sub> , kg/m <sup>3</sup>	Symbol in text
Quartz sand	0,24	5	2650	Q.1
Quartz sand	0,6	20	2540	Q.2
Silica gel	0,2	2	1100	S

TABLE 1. Characteristics of Materials

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